1. Error Bars. Two new concrete mixes are tested, resulting in the following data. Mix 1: sample size = 15 , sample mean $=4900$ psi, sample standard deviation 400 psi. Mix 2 : sample size $=31$, sample mean $=5000$ psi, sample standard deviation 150 psi.
a. What significance level is associated with a $90 \%$ Confidence level?
b. Determine the $90 \%$ Confidence Interval for each mix. State them using the proper format. Show your work. If it is appropriate to use the $Z$ distribution, do so.
c. Plot the means and error bars ( $90 \% \mathrm{Cl}$ ) in a column chart using Excel. Use proper chart formatting.
d. Do the error bars give you any statistical information regarding the difference between the mix means? If so, what?

## SOLUTION

a. $\alpha=0.1$
b. Use $\alpha / 2=0.05$ because $\mathrm{Cl}^{\prime}$ s are 2 sided. Use T for mix 1 , z for $\operatorname{mix} 2$

Mix 1:

- $U_{1}=\mu+t_{0.05,14} \cdot \frac{s}{\sqrt{n}}=4900+1.76131 \cdot \frac{400}{\sqrt{15}}=5082 \mathrm{psi}$
- $L_{1}=\mu-t_{0.05,14} \cdot \frac{s}{\sqrt{n}}=4900-1.76131 \cdot \frac{400}{\sqrt{15}}=4718 \mathrm{psi}$
- (4718 psi < $\mu<5082$ psi) with $90 \%$ Confidence

Mix 2:

- $U_{2}=\mu+t_{0.05,30} \cdot \frac{s}{\sqrt{n}}=5000+1.644854 \cdot \frac{150}{\sqrt{31}}=4956 \mathrm{psi}$
- $L=\mu-t_{0.05,30} \cdot \frac{s}{\sqrt{n}}=5000-1.644854 \cdot \frac{150}{\sqrt{31}}=5044 \mathrm{psi}$
- (4956 psi < $\mu<5044$ psi) with $90 \%$ Confidence
C.

d. The confidence interval error bars overlap; thus, one cannot say anything about the difference between the two mix means.

2. Sample Size: Determine the sample size needed to estimate the density of a new asphalt mix with a precision of $3 \mathrm{~kg} \mathrm{~m}^{-3}$ if the population standard deviation is UNKNOWN and the sample standard deviation is assumed to be $6 \mathrm{~kg} \mathrm{~m}^{-3}$. Use a significance level of 0.05 .
a. What is the confidence level?
b. What sample size is estimated using the $Z$ distribution? Can the $Z$ distribution be used? Should the T distribution be used? Explain.
c. If appropriate, what sample size is estimated using the T distribution? Document your work in a table, with sample calculations shown below the table.

## SOLUTION

a. Confidence $=1-\alpha=1-0.05=0.95$ or $95 \%$
b. Using the $Z$ distribution, $N=\left(\frac{z_{\alpha / 2} \sigma}{P}\right)^{2}=\left(\frac{1.960 \cdot 6}{3}\right)^{2}=15.4$

Because the population standard deviation is unknown and the required sample size $<30$, the $Z$ distribution should not be used.
c. Using the T distribution, $N=\left(\frac{t_{\alpha / 2, v} s}{P}\right)^{2}$, solve by trial and error.

| N Guess | $\mathrm{t}_{0.025, n-1}$ | N Calculated |
| :---: | :---: | :---: |
| 16 | 2.131 | 18.2 |
| 17 | 2.120 | 18.0 |
| 18 | 2.110 | 17.8 |
| 17.98 | 1.120 | 17.98 |

Second row sample calculation: $N$ guess $=17, t_{0.05 / 2,17-1}=t_{0.025,16}=2.120 \&\left(\frac{2.120 \cdot 6}{3}\right)^{2}=18$
Answer: $\mathrm{N}=18$.

A Solver solution gives $\mathrm{N}=17.96=18$. (Optional)
The $T$ distribution is the correct distribution, as the pop. standard deviation is unknown and the required sample size is $<30$.

